WOCOMAL

Varsity Meet #2

December 4, 2002

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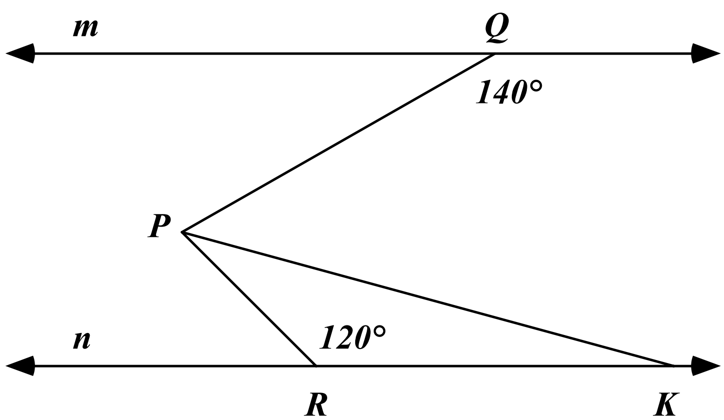
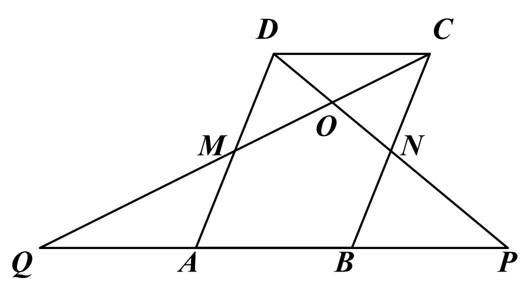
ROUND#1: **Parallel Lines & Polygons**

1. In the diagram, line ***m*** is parallel to line ***n*** and $\overline{PK}$ bisects $\angle QPR$.
   Compute the measure of $\angle PKR$.



2. The area of parallelogram ABCD is 24, and M and N are midpoints of its sides.
   Points which appear to be collinear are. Find the area of ΔPOQ.



3. A regular polygon has N sides. If the number of sides increases by 7,
   then the measure of each angle increases by 21^o. Find N.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

St. John's, Bromfield, Worcester Acad.

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ROUND#2: **Algebra 1**

1. Each day Scott earns \$3 for doing his chores or earns \$5 for doing them exceptionally well. After ten days doing his chores, Scott has earned a total of \$36. On how many of those days did Scott do his chores exceptionally well?

2. If the solution to this system is of the form $(x, y) = (a, -a)$, find a .

$$\frac{2x}{3} + \frac{y}{7} = -11$$
$$\frac{x}{7} - \frac{y}{3} = -10$$

3. Albert is A years old and John is J years old. How many years ago was John C times as old as Albert?

Answer here: 1. (1 pt.) __________days

2. (2 pts.) _________

3. (3 pts.) _________years

Bancroft, Bartlett, St. John's

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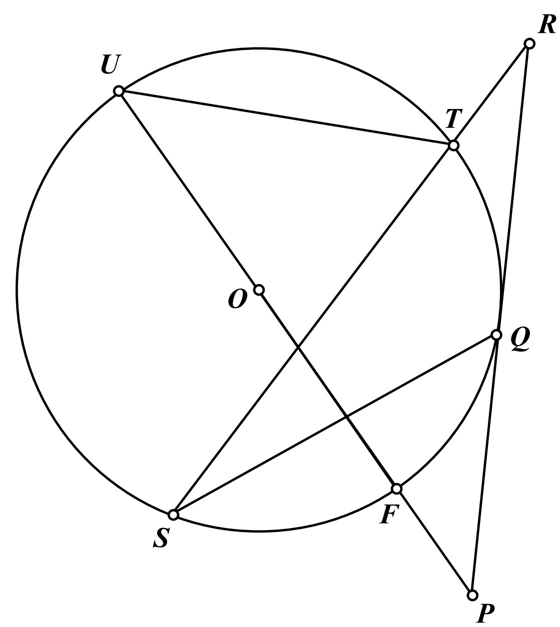
ROUND#3: **Circles & All Related Theorems**

1. In terms of π , compute the area of a sector having radius 6 and angle 100°.

2. The sides of a triangle have lengths 8, 12, and 14. Find the distance from the midpoint of the longest side to the point at which this side is tangent to the circle inscribed in the triangle.

3. $\overline{PU}$ is a secant thru center O and $\overline{PR}$ is tangent at Q. The measures of arcs $\overset{\frown}{SF}$, $\overset{\frown}{FQ}$, $\overset{\frown}{TU}$, and $\overset{\frown}{US}$ are x, $2x$, $4x$, $8x$, respectively. Find the degree-measure of $\angle UPR$.



Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________degrees

Burncoat, Hudson, Assabet

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ROUND#4: **Sequences & Series** - **<< No Calculators >>**

1. $\{a_n \mid n = 1, 2, 3, \ldots\} = \{-8, -2, \ldots\}$ is an arithmetic sequence. Compute a_{336}.

2. This continued radical converges to a number. Find that number.

$$\sqrt{63 - 2\sqrt{63 - 2\sqrt{63 - 2\sqrt{\cdots}}}}$$

3. Let $S_1 = 1$, $S_2 = 2 + 3 = 5$, $S_3 = 4 + 5 + 6 = 15$, $S_4 = 7 + 8 + 9 + 10 = 34$, ... , where S_N is the sum of the next N counting numbers not already used. Compute S_{17}.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Bartlett, Northbridge, Algonquin

ROUND#5: **Matrices & Systems of Equations** - **<< No Calculators >>**

1. The inverse of $\begin{matrix} 2 & 1 \\ 1 & -3 \end{matrix}$ equals $\frac{1}{7} \cdot \begin{matrix} a & b \\ c & d \end{matrix}$. Write the matrix $\begin{matrix} a & b \\ c & d \end{matrix}$.

2. Solve this system for all ordered pairs (x, y).

$$\begin{aligned} x^2 - 2xy + y^2 &= 25 \\ 4x + y &= 15 \end{aligned}$$

3. The determinant $\begin{vmatrix} 1 & x & y \\ 1 & y & z \\ 1 & x & z \end{vmatrix}$ can be expanded and then factored into the product of two binomials. Write this factored form.

Answer here: 1. (1 pt.) $\begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{matrix}$

2. (2 pts.) _________

3. (3 pts.) $(\_\_\_\_\_\_\_\_\_)\cdot(\_\_\_\_\_\_\_\_\_)$

Douglas, Shrewsbury , Holy Name

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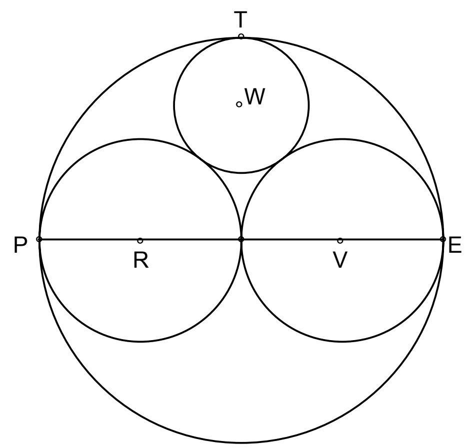
Team Round:

1. In a plane, the degree measures of the interior angles of a convex hexagon are all integers, no two of which are equal. If A is the degree measure of the third largest angle, find the minimum possible measure for A.

2. How many solutions does this equation have? $\left|\left|\left|\left|x-1\right|-2\right|-3\right|-2\right|=1$

3. $\overline{PE}$ is a diameter of a circle centered at O. Two circles are drawn with $\overline{PO}$ and $\overline{OE}$ as diameters. A fourth circle of radius 10 cm. is tangent to the three circles, as shown. Compute the diameter PE.



4. If reduced $\frac{p}{q}=\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\frac{4}{10^4}+\ldots$ where the N^{th} term of the infinite sum is $\frac{N}{10^N}$, compute $p+q$.

5. If $A=\begin{matrix} 2 & -3 \\ 4 & -5 \end{matrix}$ and $B=\begin{matrix} -8 & 6 \\ -4 & -2 \end{matrix}$, compute X if $(A\cdot X)\cdot A=B$.

6. Expand and factor: $\begin{vmatrix} 0 & a & b \\ c & 0 & a \\ b & c & 0 \end{vmatrix}+\begin{vmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{vmatrix}$.

7. Find all ordered pairs (x, y) which solve this system.

$$14x+3y=xy$$
$$7x+18y=-5xy$$

8. An old circular clock and a new digital clock keep exactly the same time in a 12-hour cycle. What is the reading on the digital clock [hh : mm : ss] when the two hands of the circular clock are together between 3 and 4 o'clock? [Six digits, please.]

9. Find the largest counting number A for which x and x^3 leave the same remainder when divided by A for any integer $x>5$.

Auburn, St. John's, Auburn, Auburn, Bromfield, QSC, Notre Dame, Quaboag, Hudson

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December 4, 2002 Team Round Varsity Meet#2

2 Points Each

Answers must be **exact** or
rounded to **three decimal** places,
except where stated otherwise.

Answers here ↓ :

1.________________________

2.________________________

3.________________________

4.________________________

5. $X = \frac{\_\_}{\_\_} \; \frac{\_\_}{\_\_}$

6.________________________

7.________________________

8. [__ __ : __ __ : __ __]

9.________________________

School:________________________________

Team#: ______

Players' Names ➔ :

#1:________________________

#2:________________________

#3:________________________

#4:________________________

#5:________________________

WOCOMAL Answers Varsity Meet #2 December 4, 2002

R#1: 1. $10°$

2. 27

3. N = 8

R#2: 1. 3 days

2. a = - 21

3. $\dfrac{CA-J}{C-1}$

R#3: 1. $10\,\pi$

2. 2

3. $50°$

R#4: 1. 2002

2. 7

3. 2,465

R#5: 1. $\begin{matrix} 3 & 1 \\ 1 & -2 \end{matrix}$

2. $(2,7)$ *and* $(4,-1)$
[Need both.]

3. $(x-y)\cdot(y-z)$ or
$(z-y)\cdot(y-x)$

Team: 1. $93°$

2. nine

3. 60 cm.

4. 91

5. $X = \begin{matrix} 1 & 3 \\ -2 & 4 \end{matrix}$

6. $b\cdot(a+c)^2$

7. $(0,0)$ *and* $(-3,7)$
[need both]

8. [03 : 16 : 21]
[six digits in order]

9. A = 6

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V2 - Solutions — Dec. 4, 2002

Round#1 1. The other angles at Q and R are 40° and 60°. So, $m\angle QPR = 40 + 60 = 100$.
Thus, $m\angle RPK = 50$ and $m\angle PKR = 10$.

2. From "midpoints" and parallels, it should be clear about areas that
$$(POQ) = (MAQ) + (MABNO) + (NBP) = (MDC) + (MABNO) + (NCD) = (ABCD) + (OCD).$$
But, $(OCD) = \frac{1}{8}(ABCD) = \frac{1}{8}(24) = 3$. So, $(POQ) = 24 + 3 = 27$.

3. Solve $\dfrac{((N+7)-2)\cdot 180}{N+7} = 21 + \dfrac{(N-2)\cdot 180}{N}$ to obtain $N = 8$.

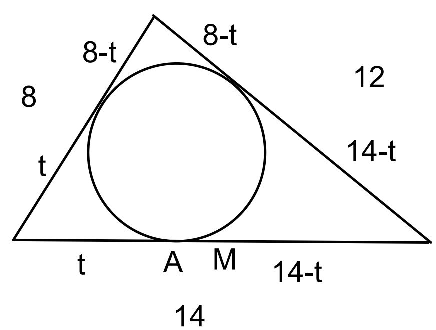
Round#2 1. Suppose he did his chores on C days and did them well on W days.
Then $3C + 5W = 36$ and $C + W = 10$. This means $W = 3$.

2. There is too much info, but it is all consistent. The easiest way is to substitute $(a, -a)$ into one equation, and find $a = -21$.

3. Suppose the answer was N years. Then, $J - N = C(A - N)$ and $N = \frac{CA-J}{C-1}$.

Round#3 1. $Area = \dfrac{100}{360} \times \pi \times 6^2 = 10\pi$.

2. From the figure,
$(8-t) + (14-t) = 12$ and $t = 5$.
So, $MA = \frac{1}{2}(14) - t = 7 - 5 = 2$.



3. Semicircle $\overset{\frown}{USF}$ measures $8x + x = 9x = 180$. So, $x = 20$. This means that $m\overset{\frown}{FQ} = 40$ and $m\overset{\frown}{QU} = 140$. So, $m\angle P = \frac{1}{2}(140 - 40) = 50$.

Round#4 1. $a_1 = -8$ and $d = 6$. So, $a_{336} = a_1 + 335d = 2002$.

2. Call the radical R, and find a duplicate of itself within itself. So, $R = \sqrt{63 - 2R}$.
The only positive solution is $R = 7$.

3. The number of numbers used from S_1 to S_{16} is $1 + 2 + 3 + ... + 16 = \frac{16\times 17}{2} = 136$.
So, $S_{17} = 137 + 138 + ... + 153 = 2465$.

Round#5 1. $\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix}^{-1} = \dfrac{1}{-7}\begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} = \dfrac{1}{7}\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. So, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}$.

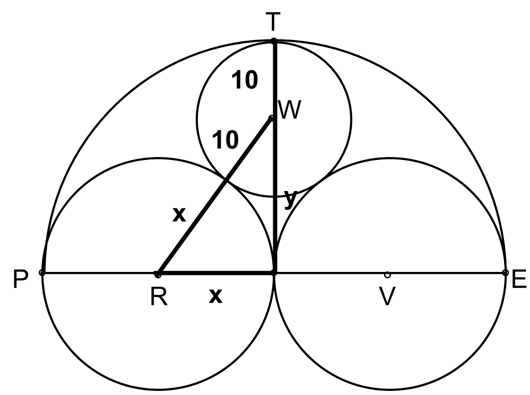
2. $x^2 - 2xy + y^2 = 25$ becomes $x - y = \pm 5$. Now solve each line with $4x + y = 15$ to obtain the results $(2, 7)$ and $(4, -1)$.

3. $\begin{vmatrix} 1 & x & y \\ 1 & y & z \\ 1 & x & z \end{vmatrix} = 1 \cdot \begin{vmatrix} y & z \\ x & z \end{vmatrix} - 1 \cdot \begin{vmatrix} x & y \\ x & z \end{vmatrix} + 1 \cdot \begin{vmatrix} x & y \\ y & z \end{vmatrix} = yz - y^2 - xz + xy = (z - y)(y - x)$.

Team 1. Suppose the angles are $X < Y < Z < A < B < C$. A will be as small as possible if B and C are as large as possible. $C = 179$ and $B = 178$. Then, $X + Y + Z + A + 178 + 179 = 720$ or $X + Y + Z + A = 363$. Since A is now the largest of these four, A will be the smallest it can be, the closer the others are to A. $\frac{363}{4} \approx 91$. So, $X = 89$, $Y = 90$, and $Z = 91$ are the largest these three can be and still be less than $A = 93°$.

2. Some one on the team must have a graphing calculator with solver; nine solutions.

3. From the figure, $y + 10 = 2x$ and $x^2 + y^2 = (x+10)^2$, and the only useful solution is $x = 15$. So, $PE = 4x = 60\,cm$.



4. A trick here is to notice that the infinite series is itself a series of infinite series. ie. $\frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \frac{4}{10^4} + ... = \frac{1}{10} + \frac{1+1}{10^2} + \frac{1+1+1}{10^3} + \frac{1+1+1+1}{10^4} + ...$

$= 0.\bar{1} + 0.0\bar{1} + 0.00\bar{1} + 0.000\bar{1} + ... = \frac{1}{9} + \frac{1}{90} + \frac{1}{900} + \frac{1}{9000} + ... = \dfrac{\frac{1}{9}}{1-\frac{1}{10}} = \dfrac{10}{81}$.

So, $p + q = 91$.

5. $X = A^{-1} \cdot \left(B \cdot A^{-1}\right) = \frac{1}{2} \cdot \begin{vmatrix} -5 & -4 \\ 3 & 2 \end{vmatrix} \cdot \begin{vmatrix} -8 & 6 \\ -4 & -2 \end{vmatrix} \cdot \frac{1}{2} \cdot \begin{vmatrix} -5 & -4 \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix}$.

6. Just do it.

7. With solver, it is easy. Without: (0,0) should be obvious. But to catch all solutions, substitute $14x + 3y$ for xy in 2nd equation. Simplify to obtain $-7x = 3y$, use this to substitute to eliminate a variable and find (-3,7).

8. First find the angle α thru which the hour hand turns past 3 o'clock. Since the minute hand moves 12 times as fast, $90 + \alpha = 12\alpha$ and $\alpha = \frac{90}{11}$ *degrees*. $1^o = \frac{1}{6}$ *minute*. So, α represents $\frac{90}{11} \times \frac{1}{6} = \frac{15}{11} = 1\frac{4}{11}$ *minutes* past 15 minutes past 3 o'clock. We thus know that the hour will read 03 and the minute will read 16. To obtain the seconds, convert the remaining $\frac{4}{11} \times 60 = 21.8$; but the .8 will not show. The answer is 03 : 16 : 21.

9. x is an integer, and x^3 and x both have the same remainder upon division by some number A if and only if their difference $x^3 - x$, which always factors into $(x-1)(x)(x+1)$, always contains three successive integers. These three will always contain at least one factor of 2 and one factor of 3, making it divisible by 6. And 6 is the only higher divisor about which this claim can always be made.